TIMED ALTERNATING-TIME TEMPORAL LOGIC

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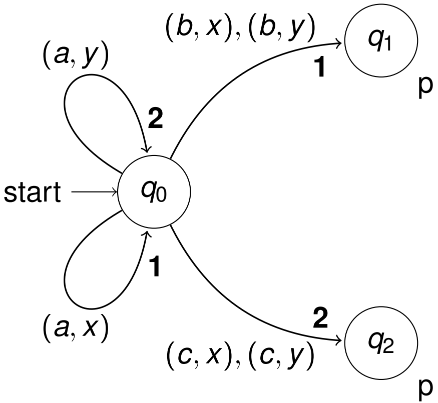
Outline

- **Base framework**: Alternating-Time Temporal Logic extended with discrete, non-Zeno time [Markey et. al]
- **Our results**:
  - Time history is irrelevant.
  - Time is irrelevant, unless we want strict punctuality: strategies based on the number of visits at a location.
  - Two actions per location are sufficient to implement any counting strategy, unless strict punctuality is needed.

A **Tight Durational Concurrent Game Structure** is a 7–tuple $\mathcal{A} = (\mathit{Agents}, \Sigma, \mathcal{Q}, \mathcal{AP}, \mathcal{L}, \mathit{pr}, t)$, where:

- *Agents* is a finite set of all the *agents*,
- Σ is a finite set of *actions*,
- $\mathcal{Q}$ is a finite set of *locations*,
- $\mathcal{AP}$ is a set of *atomic propositions*,
- $\mathcal{L} \colon \mathcal{Q} \to \mathcal{P}(\mathcal{AP})$ is a *location labeling function*,
- $\mathit{pr} \colon \mathit{Agents} \times \mathcal{Q} \to \mathcal{P}(\Sigma) \setminus \{\emptyset\}$ is a *protocol function*,
- $t \colon \mathcal{Q} \times \Sigma^{|\mathit{Agents}|} \to \mathcal{Q} \times \mathbb{N}_{+}$ is a *transition function*.

Models, ct'd



We model runs in a state/time space: $\mathcal{S} := Q \times \mathbb{N}$, e.g.:

$$(q_0, 0) \xrightarrow{(a,y)} (q_0, 2) \xrightarrow{(a,x)} (q_0, 3) \xrightarrow{(c,y)} (q_2, 5)$$

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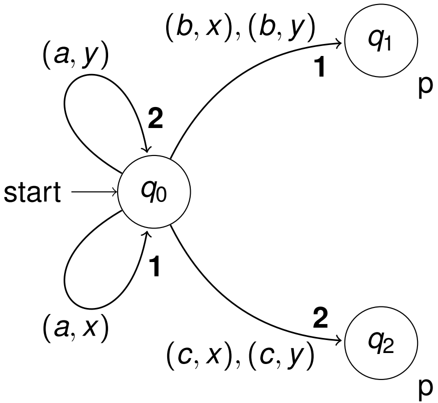

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Strategies

Notations: let $q \in \mathcal{Q}$, $s \in \mathcal{S}$ and $\pi \in \mathcal{S}^+ \cup \mathcal{S}^\omega$.

- $lc(s)$ **and** $tm(s)$: location and time, resp., of s,
- $\pi(i)$: i**–th state** of π,
- π_i: **prefix of** π of length i,
- π^i: **postfix of** π starting from $\pi(i)$,
- for finite π:
  - π_F: **final state** of π,
  - $\#_F(\pi)$: number of states of π whose location is $lc(\pi_F)$.

…count how many times the final location appears along π, e.g.:

$$\pi = ((q_0, 0), (q_0, 2)),$$
$$\pi' = ((q_0, 0), (q_0, 2), (q_0, 3)),$$
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$\#_F(\pi) = 2$, $\#_F(\pi') = 3$, $\#_F(\pi'') = 1$.

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Define the following types of strategies for $a \in Agents$:

TIMED PERFECT RECALL STRATEGIES (Σ_{T})
Functions $\sigma_a\colon \mathcal{S}^+ \to \Sigma$ s.t. $\forall_{\pi \in \mathcal{S}^+} \sigma_a(\pi) \in pr_a(lc(\pi_F))$.

(Intuition: no constraints, apart from the protocol)

TIMED MEMORYLESS STRATEGIES (Σ_{t})
Strategies $\sigma_a \in \Sigma_{\mathrm{T}}$ s.t., for each $\pi, \pi' \in \mathcal{S}^+$, if $\pi_F = \pi'_F$, then $\sigma_a(\pi) = \sigma_a(\pi')$.

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UNTIMED MEMORYLESS STRATEGIES (Σ_{R})
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THRESHOLD STRATEGIES ($\Sigma_{\#_n}$)

A counting strategy $\sigma_a \in \Sigma_{\#}$ is called *n*–**threshold** for some $n \in \mathbb{N}_+$ iff for each location $q \in Q$ there are:

- actions $act_1, \ldots, act_{n+1} \in \Sigma$, and
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s.t. for all $1 \leq j \leq n+1$: $\sigma_a^{\#}(q, k) = act_j$ if $k \in I_j$.

(Much-needed intuition: e.g., a counting strategy is 2–threshold if for any location $q \in Q$ there are ***three*** *actions act_1, act_2, act_3 s.t. first only act_1 is used when q is visited, then only act_2, and finally only act_3, ad infinitum.*

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Strategies, ct'd

Let $A \subseteq \mathit{Agents}$.

- **A joint strategy** σ_A for A is a tuple of strategies, one per agent $a \in A$.

  Notation: if $A = \{a_1, \ldots, a_k\}$ for some $k \in \mathbb{N}$ and $\sigma_A = (\sigma_{a_1}, \ldots, \sigma_{a_k})$ is a joint strategy for A, then, for each $i \in \mathbb{N}$ and $\pi \in \mathcal{S}^\omega$, denote $\sigma_A(\pi_i) := (\sigma_{a_1}(\pi_i), \ldots, \sigma_{a_k}(\pi_i))$.

- **The outcome** of σ_A in state $s \in \mathcal{S}$ is the set $out(s, \sigma_A) \subseteq \mathcal{S}^\omega$ s.t. $\pi \in out(s, \sigma_A)$ iff $\pi(0) = s$ and, for each $i \in \mathbb{N}$, there is $act' \in pr_{\overline{A}}(lc(\pi(i)))$ s.t. $\mathcal{E}(\pi(i), (\sigma_A(\pi_i), act')) = \pi(i+1)$.

Intuition: when coalition A follows σ_A, then at every state it selects actions according to the joint strategy while the remaining agents can choose anything they wish.

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$$\phi ::= \mathsf{p} \mid \neg\phi \mid \phi \vee \phi \mid \langle\langle A \rangle\rangle X \phi \mid \langle\langle A \rangle\rangle \phi U_{\sim\eta} \phi \mid \langle\langle A \rangle\rangle \phi R_{\sim\eta} \phi$$

where $\mathsf{p} \in \mathcal{AP}$, $A \subseteq Agents$, $\sim \in \{\leq, =, \geq\}$, and $\eta \in \mathbb{N}$.

We interpret $\langle\langle A \rangle\rangle \psi$ as "coalition A has a strategy to enforce ψ", X stands for "at the next state", U for "until", and R for "release".

Derived modalities: F (*"in the future"*) and G (*"globally"*):
$\langle\langle A \rangle\rangle F_{\sim\eta} \phi := \langle\langle A \rangle\rangle \top U_{\sim\eta} \phi$, $\langle\langle A \rangle\rangle G_{\sim\eta} \phi := \langle\langle A \rangle\rangle \bot R_{\sim\eta} \phi$.

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Exemplary properties:

- $\langle\langle A\rangle\rangle F_{=13}$**finish**: *"Coalition A* **has a strategy to enforce** *that* **finish** *is reached at precisely* 13 *time units".*
- $\langle\langle A\rangle\rangle G_{\geq 42}$**safe**: *"Coalition A* **has a strategy to enforce** *that* **safe** *holds always after reaching* 42 *time units".*

For each type of strategy, we define corresp. satisfaction relation identified by resp. superscript; e.g. $\models_R$ corresponds to Σ_{R}.

SATISFACTION

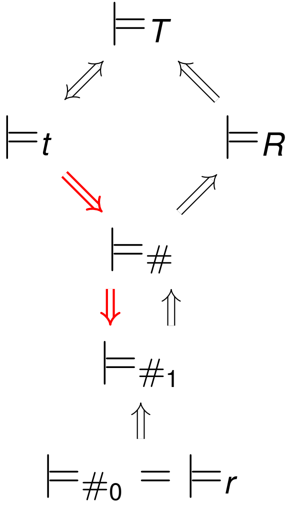
$s \models_Y \langle\langle A \rangle\rangle \psi$: *There is a strategy* $\sigma_A \in \Sigma_Y$ *for* A *s.t.* ψ *holds* **along each outcome** $\pi \in out(s, \sigma_A)$.

Satisfaction over outcomes:

- $\pi \models X\phi$ iff $\pi(1) \models \phi$,
- $\pi \models \phi U_{\sim\eta} \psi$ iff $\pi(i) \models \psi$ for some i s.t. $tm(\pi_i) \sim \eta$ and $\pi(j) \models \phi$ for all $j < i$,
- $\pi \models \phi R_{\sim\eta} \psi$ iff $tm(\pi_i) \sim \eta$ implies that $\pi(i) \models \psi$ or $\pi(j) \models \phi$ for some $j < i$,

...the boolean operations are as usual.

Hierarchy of satisfactions



Red implications hold only for $\text{TATL}_{\leq,\geq}$, i.e., formulae without equalities.

Key implications: timed strategies and memory

(1) Timed strategies do not need memory

For each $q \in \mathcal{Q}$ and $\phi \in$ TATL, we have $q \models_T \phi$ iff $q \models_t \phi$.
(so we omit subscript and write $\models$)

Easy result: time limit

Let $\langle\langle A \rangle\rangle \psi \in$ TATL. If $c \in \mathbb{N}$ is the greatest integer present in ψ, then there is no need to track time after it exceeds c.

More formally: if $\sigma_A \in \Sigma_\mathrm{T}$ implements $\langle\langle A \rangle\rangle \psi$, then there is a reduction σ'_A of $\sigma_A \in \Sigma_\mathrm{T}$ s.t. $\forall_{q \in \mathcal{Q}} \forall_{t \geq c}\ \sigma'_A(q, t) = \sigma'_A(q, c+1)$.

Key implications: time vs order

(2) TRUE IN COUNTING $\implies$ TRUE IN TIMED

For each $q \in \mathcal{Q}$ and $\phi \in$ TATL, if $q \models_{\#} \phi$, then $q \models \phi$.

Easy: just disregard the clock in memoryful outcomes.

Key implications: time vs order, ct'd

Recall $\text{TATL}_{\leq,\geq}$: subset of TATL with only $\leq, \geq$ allowed, e.g., $\langle\langle A\rangle\rangle G_{\geq 42}$**safe**, but not $\langle\langle A\rangle\rangle F_{=13}$**finish**.

(3) TRUE IN TIMED $\implies$ TRUE IN COUNTING

For each $q \in \mathcal{Q}$ and $\phi \in \text{TATL}_{\leq,\geq}$, if $q \models \phi$, then $q \models_{\#} \phi$.

Cannot be extended to TATL, see next slide.

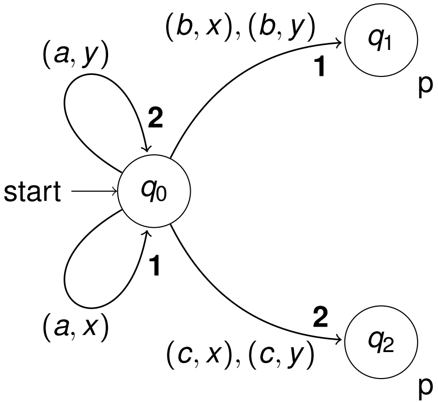
Recall $\text{TATL}_{\leq,\geq}$: subset of TATL with only $\leq, \geq$ allowed, e.g., $\langle\langle A\rangle\rangle G_{\geq 42}$**safe**, but not $\langle\langle A\rangle\rangle F_{=13}$**finish**.

(3) TRUE IN TIMED $\Longrightarrow$ TRUE IN COUNTING

For each $q \in \mathcal{Q}$ and $\phi \in \text{TATL}_{\leq,\geq}$, if $q \models \phi$, then $q \models_{\#} \phi$.

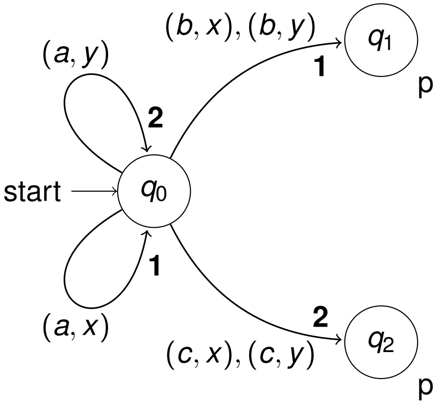
Cannot be extended to TATL, see next slide.

Key implications: time vs order, ct'd



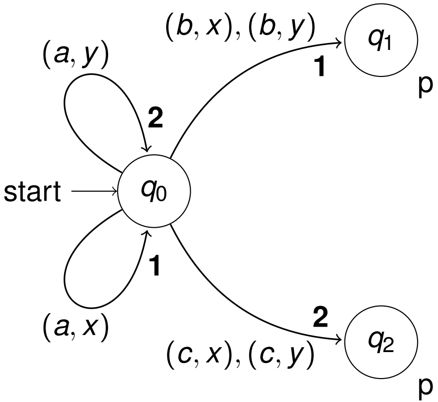
Observe: $q_0 \models \langle\langle 1 \rangle\rangle F_{=5}\mathrm{p}$, but $q_0 \not\models_{\#} \langle\langle 1 \rangle\rangle F_{=5}\mathrm{p}$, as there is no counting strategy that allows for deciding when to leave q_0 for a location labeled with p, and which branch to take in order to reach the target in 5 time units.

Key implications: time vs order, ct'd



Observe: $q_0 \models \langle\langle 1 \rangle\rangle F_{=5}\mathsf{p}$, but $q_0 \not\models_{\#} \langle\langle 1 \rangle\rangle F_{=5}\mathsf{p}$, as there is no counting strategy that allows for deciding when to leave q_0 for a location labeled with p, and which branch to take in order to reach the target in 5 time units.

Key implications: time vs order, ct'd



Observe: $q_0 \models \langle\langle 1 \rangle\rangle F_{=5}\mathrm{p}$, but $q_0 \not\models_{\#} \langle\langle 1 \rangle\rangle F_{=5}\mathrm{p}$, as there is no counting strategy that allows for deciding when to leave q_0 for a location labeled with p, and which branch to take in order to reach the target in 5 time units.

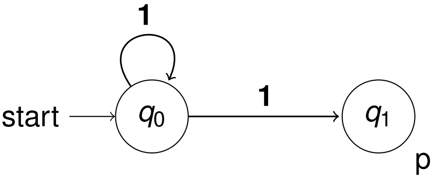
Key implications: counting up to...

(4) The threshold for $\mathsf{TATL}_{\leq,\geq}$ is 1

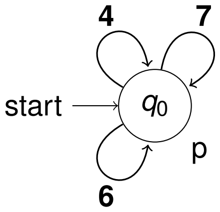
For each $q \in \mathcal{Q}$ and $\phi \in \mathsf{TATL}_{\leq,\geq}$, if $q \models_{\#} \phi$, then $q \models_{\#_1} \phi$.

All modalities apart from $U_{\geq\eta}$ need only one action, while $U_{\geq\eta}$ needs two.

... and cannot be lowered



- $q_0 \models_{\#_1} \langle\langle 1 \rangle\rangle F_{\geq 5}$: loop four times and jump ahead
- $q_0 \not\models_{\#_0} \langle\langle 1 \rangle\rangle F_{\geq 5}$: loop forever, or jump too early



(5) THERE IS NO THRESHOLD FOR TATL

$\langle\langle 1\rangle\rangle F_{=17}$p: **three distinct actions** needed to sum up to exactly 17 time units.

This can be extended to an arbitrary number of actions using Ł. Mikulski's sequence: $(10)^n + (1 \ldots 2^n)_{(binary)}$.

Our results, once again:

- Time history is irrelevant.
- Time is irrelevant, unless we want strict punctuality: strategies based on the number of visits at a location.
- Two actions per location are sufficient to implement any counting strategy, unless strict punctuality is needed.

Future work:

- What changes for incomplete knowledge?
- Are there any practical implications?
- Extension to TATL*.

Thank you!