Zbiory przybliżone i systemy tranzycyjne (Rough Sets and Transition Systems)

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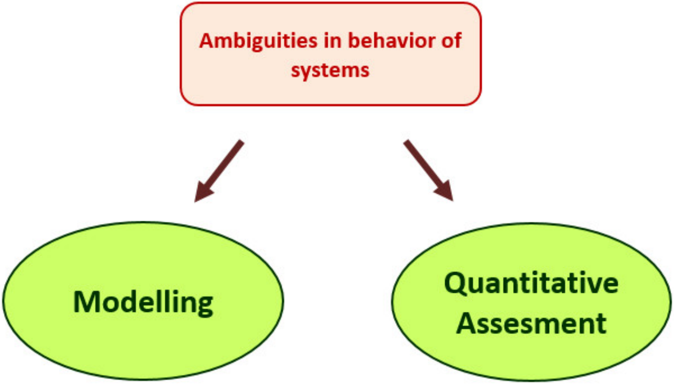
The main goal of the reserach:



Transition systems
are used to describe behavior of systems with distinguished states and transitions between states.

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Rough sets
are an appropriate tool to deal with rough (**ambiguous**, imprecise) concepts in the universe of discourse.

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There are cases in which behavior of systems is characterized by some ambiguities of transition between states.

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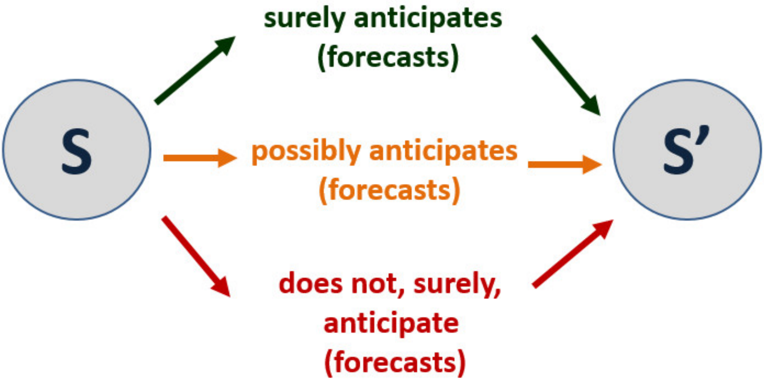
Approaches, based on rough sets, to model and assess ambiguities of transition between states, have been proposed

Introduction



Rough Set

- $U \neq \emptyset$ - a finite set of objects we are interested in.
- R - any equivalence relation over U.
- $[u]_R$ - an equivalence class of any $u \in U$.

With each $X \subseteq U$ and R, we associate two subsets:

- the R-lower approximation of X: $\underline{R}(X) = \{u \in U : [u]_R \subseteq X\}$,
- the R-upper approximation of X: $\overline{R}(X) = \{u \in U : [u]_R \cap X \neq \emptyset\}$.

Pawlak 1991

Pawlak, Z.: Rough Sets. Theoretical Aspects of Reasoning about Data. Kluwer Academic Publishers, Dordrecht (1991).

Basic Definitions

Standard Set Inclusion

Let U be the universe and $A, B \subseteq U$. The standard set inclusion is defined as

$$A \subseteq B \text{ if and only if } \forall_{u \in A} \; u \in B.$$

Majority Set Inclusion

Let U be the universe, $A, B \subseteq U$, and $0 \leq \beta < 0.5$. The majority set inclusion is defined as

$$A \overset{\beta}{\subseteq} B \text{ if and only if } 1 - \frac{card(A \cap B)}{card(A)} \leq \beta,$$

Variable Precision Rough Set Model (VPRSM)

By replacing the standard set inclusion with the majority set inclusion in definitions of approximations, we obtain the following two subsets:

- the R_β-lower approximation of X:

  $\underline{R}^\beta(X) = \{u \in U : [u]_R \overset{\beta}{\subseteq} X\}$,
- the R_β-upper approximation of X:
  $\overline{R}^\beta(X) = \{u \in U : 1 - \frac{card([u]_R \cap X)}{card([u]_R)} < 1 - \beta\}$.

Ziarko 1993

Ziarko, W.: Variable precision rough set model. Journal of Computer and System Sciences 46(1), 39-59 (1993).

Transition system

A transition system is a quadruple $TS = (S, E, T, S_{init})$, where:

- S is the non-empty set of states,
- E is the set of events,
- $T \subseteq S \times E \times S$ is the transition relation,
- $S_{init} \subseteq S$ is the set of initial states.

Basic Definitions

Timed Transition System

Timed Transition Systems $TTS = (S, E, T, S_{init}, d_{min}, d_{max})$ consists of:

- the underlying transition system $TS = (S, E, T, S_{init})$,
- the minimal delay function (a lower bound) $d_{min} : E \to N$ assigning a nonnegative integer to each event,
- the maximal delay function (an upper bound) $d_{max} : E \to N \cup \{\infty\}$ assigning a nonnegative integer or infinity to each event.

We assume, for timed transition systems, that the events may occur only at discrete time instants. Therefore, whenever time instant t is used, it means that $t \in \{t_0, t_1, t_2, \dots\}$.

Direct Successors and Predecessors for States in Timed Transition Systems

For each state $s \in S$ in the timed transition system TTS, we can determine its direct successors and predecessors. Let

- $Post_t(s, e) = \{s' \in S : (s, e, s') \in T \wedge d_{min}(e) \leq t \leq d_{maxn}(e)\}$,
- $Pre_t(s, e) = \{s' \in S : (s', e, s) \in T \wedge d_{min}(e) \leq t \leq d_{max}(e)\}$,

then the set $Post_t(s)$ of all direct successors of the state $s \in S$ at t is given by

$$Post_t(s) = \bigcup_{e \in E} Post_t(s, e)$$

and the set $Pre_t(s)$ of all direct predecessors of the state $s \in S$ at t is given by

$$Pre_t(s) = \bigcup_{e \in E} Pre_t(s, e).$$

Lower Predecessor Anticipation in Timed Transision Systems

Let $TTS = (S, E, T, S_{init}, d_{min}, d_{max})$ be a timed transition system and $X \subseteq S$. The lower predecessor anticipation $\underline{Pre}_t(X)$ of X at the time instant t is given by

$$\underline{Pre}_t(X) = \{s \in S : Post_t(s) \neq \emptyset \wedge Post_t(s) \subseteq X\}.$$

The lower predecessor anticipation $\underline{Pre}_t(X)$ consists of all states from which TTS surely goes to the states in X as results of any events occurring at these states at the time instant t.

Upper Predecessor Anticipation in Timed Transision Systems

Let $TTS = (S, E, T, S_{init}, d_{min}, d_{max})$ be a timed transition system and $X \subseteq S$. The upper predecessor anticipation $\overline{Pre}_t(X)$ of X at the time instant t is given by

$$\overline{Pre}_t(X) = \{s \in S : Post_t(s) \cap X \neq \emptyset\}.$$

The upper predecessor anticipation $\overline{Pre}_t(X)$ consists of all states from which TTS possibly goes to the states in X as results of some events occurring at these states at the time instant t. It means that TTS can also go at t to the states from outside X.

β-Lower Predecessor Anticipation in Timed Transision Systems

The β-lower predecessor anticipation $\underline{Pre}_t^{\beta}(X)$ of X at the time instant t is given by

$$\underline{Pre}_t^{\beta}(X) = \{s \in S : Post_t(s) \neq \emptyset \wedge Post_t(s) \overset{\beta}{\subseteq} X\}.$$

The β-lower predecessor anticipation of X at t consists of each state from which TTS goes, in most cases (i.e., in terms of the majority set inclusion) to the states in X as results of events occurring at these states at the time instant t.

Rough Anticipation

Continuous Strict Anticipator

If

$$\forall_{t \in \{t_0, t_1, t_2, \dots\}} s \in \underline{Pre}_t(X),$$

then s is said to be a continuous strict anticipator of states from X. It means that s always anticipates (i.e., at each time instant) states from X.

Interim Strict Anticipator

If s is not a continuous strict anticipator of states from X, but

$$\exists_{t \in \{t_0, t_1, t_2, \dots\}} s \in \underline{Pre}_t(X),$$

then s is said to be an interim strict anticipator of states from X. It means that s sometimes (not always) anticipates states from X.

Continuous Quasi-Anticipator

If s is not a continuous and interim strict anticipator of states from X, but

$$\forall_{t \in \{t_0, t_1, t_2, \dots\}} s \in \underline{Pre}_t^{\beta}(X),$$

then s is said to be a continuous quasi-anticipator of states from X.

Interim Quasi- Anticipator

If s is not a continuous and interim strict anticipator and continuous quasi-anticipator of states from X, but

$$\exists_{t \in \{t_0, t_1, t_2, \dots\}} s \in \underline{Pre}_t^{\beta}(X),$$

then s is said to be an interim quasi-anticipator of states from X.

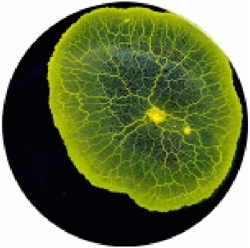
PhyChip Project

PhyChip

- PhyChip was a collaborative project funded by the Seventh Framework Programme (FP7).
- The main goal of the project was to implement programmable amorphous biological computers in plasmodium of *Physarum polycephalum*.

http://www.phychip.eu/

Physarum Polycephalum



- *Physarum polycephalum* is a one-cell organism.
- In the phase of plasmodium, it looks like an amorphous giant amoeba with networks of protoplasmic tubes.
- A *Physarum* machine is a biological computing device implemented in the plasmodium of *Physarum polycephalum*.

Modeling Propagation of Plasmodium

A structure of the *Physarum* machine

$$\mathcal{PM} = (P, A, R),$$

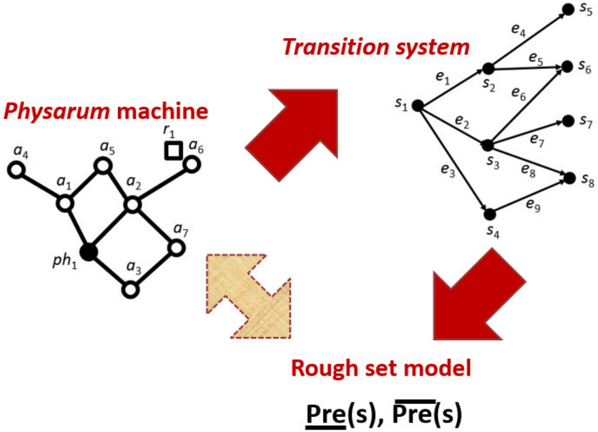
where:

- $P = \{ph_1, ph_2, \ldots, ph_k\}$ is a set of original points of plasmodium.
- $A = \{a_1, a_2, \ldots, a_m\}$ is a set of attractants.
- $R = \{r_1, r_2, \ldots, r_n\}$ is a set of repellents.

A dynamics (behavior) of $\mathcal{PM}$ over time

The family $V = \{V^t\}_{t \in \{t_0, t_1, t_2, \ldots\}}$ of the sets of protoplasmic veins propagated by the plasmodium, where $V^t = \{v_1^t, v_2^t, \ldots, v_{card(V^t)}^t\}$ is the set of all protoplasmic veins of the plasmodium present at time instant t in $\mathcal{PM}$. Each vein $v_i^t \in V^t$, where $i = 1, 2, \ldots, card(V^t)$, is the unordered pair $\{\pi_j^t, \pi_k^t\}$ such that $\pi_j^t \in P \cup A$ and $\pi_k^t \in P \cup A$.

Establishing or annihilating the protoplasmic veins can be controlled by means of attractants or repellents (their activation or deactivation)!


Physarum machine
r_1
a_4
a_5
a_6
a_1
a_2
a_7
ph_1
a_3
Transition system
s_1
s_2
s_3
s_4
s_5
s_6
s_7
s_8
e_1
e_2
e_3
e_4
e_5
e_6
e_7
e_8
e_9
Rough set model
$\underline{\mathrm{Pre}}(s)$, $\overline{\mathrm{Pre}}(s)$

Physarum machine $\longrightarrow$ Transition system

INPUT $\mathcal{PM} = (P, A, R)$ - the *Physarum* machine with the family $V = \{V^t\}_{t \in \{t_0, t_1, t_2, \dots\}}$ of the sets of protoplasmic veins describing behavior of $\mathcal{PM}$.

OUTPUT $TTS(\mathcal{PM}) = (S, E, T, S_{init}, d_{min}, d_{max})$ - a timed transition system modeling behavior of $\mathcal{PM}$.

- $\sigma : P \cup A \rightarrow S$ assigning a state to each original point of plasmodium as well as to each attractant,
- $\tau : \left(\bigcup_{t \in \{t_0, t_1, t_2, \dots\}} V_t \right) \rightarrow T$ assigning a transition to each protoplasmic vein,
- $\epsilon : \left(\bigcup_{t \in \{t_0, t_1, t_2, \dots\}} V_t \right) \rightarrow E$ assigning an event to each protoplasmic vein,
- $\iota : P \rightarrow S_{init}$ assigning an initial state to each original point of plasmodium.
- d_{min} and d_{max} are determined on the basis of V.

Transition system $\longrightarrow$ Rough set model

INPUT $TTS(\mathcal{PM}) = (S, E, T, S_{init}, d_{min}, d_{max})$ - a timed transition system modeling behavior of the *Physarum* machine $\mathcal{PM}$.

OUTPUT $RSM(\mathcal{PM}) = (S, S_{init}, \{(\underline{Pre}_t(s), \overline{Pre}_t(s))\}_{s \in (S-I), t \in \{t_0, t_1, t_2, \dots\}})$ - a rough set model of behavior of $\mathcal{PM}$.

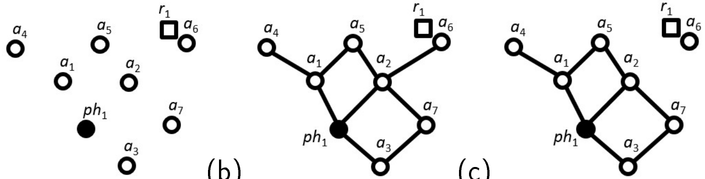
- S - the set of states defined in $TTS(\mathcal{PM})$,
- S_{init} - the set of initial states defined in $TTS(\mathcal{PM})$,
- $\{(\underline{Pre}_t(s), \overline{Pre}_t(s))\}_{s \in (S-I), t \in \{t_0, t_1, t_2, \dots\}}$ - the family of lower and upper predecessor anticipations of states, excluding initial ones.

Let us consider an exemplary *Physarum* machine $\mathcal{PM} = \{P, A, R\}$, where:

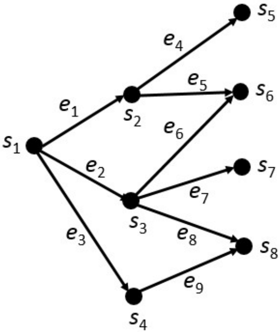
- $P = \{ph_1\}$,
- $A = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$,
- $R = \{r_1\}$,

at three time instants t_0, t_1, and t_2.

(a) (b) (c)



- A timed transition system model of $\mathcal{PM}$:



$d_{min}(e_1) = d_{min}(e_2) = d_{min}(e_3) = d_{min}(e_4) = d_{min}(e_5) = d_{min}(e_6) = d_{min}(e_7) = d_{min}(e_8) = d_{min}(e_9) = 0$, $d_{max}(e_1) = d_{max}(e_2) = d_{max}(e_3) = d_{max}(e_4) = d_{max}(e_5) = d_{max}(e_6) = d_{max}(e_8) = d_{max}(e_9) = \infty$, and $d_{max}(e_7) = 4$.

Modeling Propagation of Plasmodium

Let us assume that we are interested in the set $X = \{s_5, s_6, s_8\}$ of goal states and $\beta = 0.5$. For X and $t < 5$, we obtain:

- $\underline{Pre}_t(X) = \{s_2, s_4\}$,
- $\overline{Pre}_t(X) = \{s_2, s_3, s_4\}$,
- but $\underline{Pre}_t^{0.5}(X) = \{s_2, s_3, s_4\}$.

For X and $t \geq 5$, we obtain:

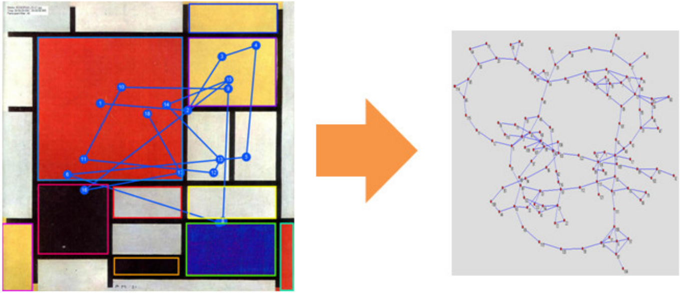
- $\underline{Pre}_t(X) = \{s_2, s_3, s_4\}$,
- $\overline{Pre}_t(X) = \{s_2, s_3, s_4\}$.

It means that:

- s_2 and s_4 are **continuous strict anticipators** of states from X,
- s_3 is an **interim strict anticipator** of states from X,
- but also s_3 is a **continuous quasi-anticipator** of states from X.

Eye-tracking Sequences

Eye-tracking sequences can be considered in terms of complex networks/transitions systems.



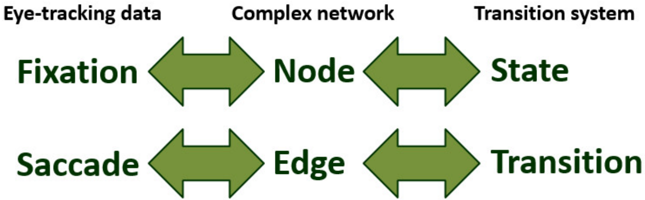
Eye-tracking Sequences

Fixation

- when the eye gaze pauses in a certain position.

Saccade

- when the eye gaze moves to another position.



Rough Sets and Complex Networks

Transformation of a sequence of eye-tracking data (a sequence of points in two-dimensional space) into an undirected graph representing a complex network.

Identification of object components in a visual stimulus and assignment of nodes of the complex network to each identified component.

Assessment of the cohesion of saccade connections between object components identified in a visual stimulus.

In the set of nodes of the complex network, we can distinguish:

- Nodes corresponding to object components.
- Nodes corresponding to eye-tracking points not covered by any object component. Such nodes are called insignificant nodes.

Let $O = \{o_1, o_2, \ldots, o_v\}$ be a set of all object components identified in the visual stimulus. We use the following notation:

- $\mathcal{N} = \{N_{o_1}, N_{o_2}, \ldots, N_{o_v}\}$ denotes a family of sets of nodes corresponding to object components.
- $N^{\ominus}$ denotes a set of insignificant nodes.

Inter-component saccade neighborhood

For each node $n \in N_{o_1} \cup N_{o_2} \cup \cdots \cup N_{o_v}$, we define its inter-component saccade neighborhood:

$$ICSN(n) = \{n' : (n, n') \in E \wedge \underset{o \in O}{\exists} (n' \in N_o \wedge n \notin N_o)\}.$$

Lower approximation

The lower approximation $\underline{ICSN}(o_i \to o_j)$ of the inter-component saccade neighborhood, from o_i to o_j, is given by:

$$\underline{ICSN}(o_i \to o_j) = \\ = \{n \in N_{o_i} : ICSN(n) \neq \emptyset \wedge ICSN(n) \subseteq N_{o_j}\}.$$

The lower approximation $\underline{ICSN}(o_i \to o_j)$ of the inter-component saccade neighborhood consists of all nodes N_{o_i} which are connected by inter-component edges with nodes from N_{o_j} only.

Upper approximation

The upper approximation $\overline{ICSN}(o_i \to o_j)$ of the inter-component saccade neighborhood, from o_i to o_j, is given by:

$$\overline{ICSN}(o_i \to o_j) = \{n \in N_{o_i} : ICSN(n) \cap N_{o_j} \neq \emptyset\}.$$

The upper approximation $\overline{ICSN}(o_i \to o_j)$ of the inter-component saccade neighborhood consists of all nodes N_{o_i} which are connected at least by one inter-component edge with nodes from N_{o_j}.

Accuracy of approximation

The accuracy of approximation of the inter-component saccade neighborhood can be defined analogously to the accuracy of approximation in rough set theory, i.e.:

$$\alpha_{ICSN}(o_i \to o_j) = \frac{card(\underline{ICSN}(o_i \to o_j))}{card(\overline{ICSN}(o_i \to o_j))}.$$

- We treat $\alpha_{ICSN}(o_i \to o_j)$ as a measure of the cohesion of saccade connections from the object component o_i to the object component o_j.
- If $\alpha_{ICSN}(o_i \to o_j) = 1$, then the connections are the most coherent ones.

β-lower approximation

Let $0 \leq \beta < 0.5$. The β-lower approximation $\underline{ICSN}(o_i \to o_j)$ of the inter-component saccade neighborhood, from o_i to o_j, is given by:

$$\underline{ICSN}^{\beta}(o_i \to o_j) =$$
$$= \{n \in N_{o_i} : ICSN(n) \neq \emptyset \wedge ICSN(n) \overset{\beta}{\subseteq} N_{o_j}\}.$$

The β-lower approximation of the inter-component saccade neighborhood $\underline{ICSN}(o_i \to o_j)$ consists of all nodes N_{o_i} which are connected by inter-component edges, in most cases (i.e., in terms of the majority set inclusion), with nodes from N_{o_j}.

β-upper approximation

The β-upper approximation $\overline{ICSN}(o_i \rightarrow o_j)$ of the inter-component saccade neighborhood, from o_i to o_j, is given by:

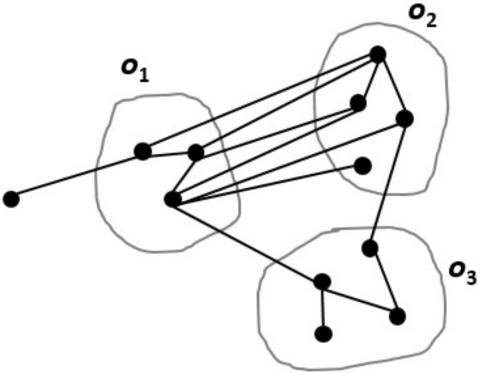
$$\overline{ICSN}^{\beta}(o_i \rightarrow o_j) = \\ = \left\{ n \in N_{o_i} : \frac{card(ICSN(n) \cap N_{o_j})}{card(ICSN(n))} > \beta \right\}.$$

Relaxed measure of the cohesion

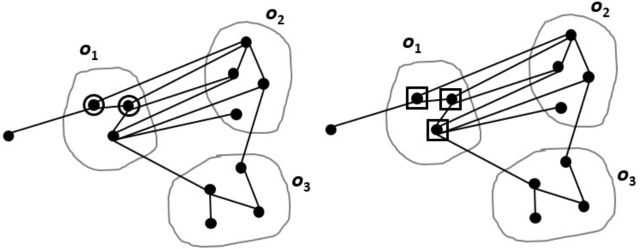
A relaxed measure of the cohesion of saccade connections from the object component o_i to the object component o_j has the form:

$$\alpha^{\beta}_{ICSN}(o_i \to o_j) = \frac{card(\underline{ICSN}^{\beta}(o_i \to o_j))}{card(\overline{ICSN}^{\beta}(o_i \to o_j))}.$$

An example of the the fragment of a complex network over the visual stimulus:



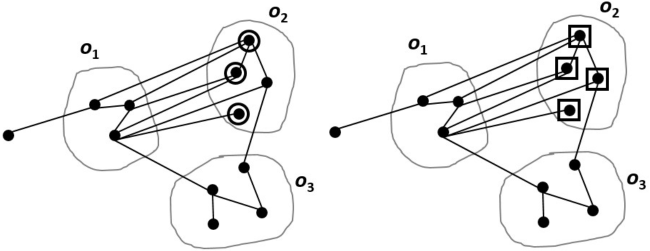
The cohesion of saccade connections, from o_1 to o_2:



$$\alpha_{ICSN}(o_1 \to o_2) = \frac{2}{3}.$$

- Nodes belonging to the lower approximation are marked with circles.
- Nodes belonging to the upper approximation are marked with rectangles.

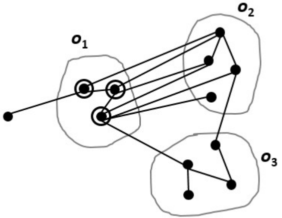
The cohesion of saccade connections, from o_2 to o_1:



$$\alpha_{ICSN}(o_2 \to o_1) = \frac{3}{4}.$$

- Nodes belonging to the lower approximation are marked with circles.
- Nodes belonging to the upper approximation are marked with rectangles.

In case of the VPRSM approach (i.e., more relaxed case), for $\beta = 0.25$, the cohesion of saccade connections, from o_1 to o_2:



$$\alpha^{0.25}_{ICSN}(o_1 \rightarrow o_2) = 1.$$

because:

$$\overline{ICSN}^{0.25}(o_1 \rightarrow o_2) = \underline{ICSN}^{0.25}(o_1 \rightarrow o_2).$$

- Nodes belonging to the β-lower approximation are marked with circles.

Conclusions

- We have shown that rough set theory can be used to describe some ambiguities in behavior of systems described by transition systems.
- We will extend the spectrum of measures by applying various rough set approaches.

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